

## Article

# Certain Notions of Neutrosophic Topological $K$ -Algebras

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**Abstract:** The concept of neutrosophic set from philosophical point of view was first considered by Smarandache. A single-valued neutrosophic set is a subclass of the neutrosophic set from a scientific and engineering point of view and an extension of intuitionistic fuzzy sets. In this research article, we apply the notion of single-valued neutrosophic sets to  $K$ -algebras. We introduce the notion of single-valued neutrosophic topological  $K$ -algebras and investigate some of their properties. Further, we study certain properties, including  $C_5$ -connected, super connected, compact and Hausdorff, of single-valued neutrosophic topological  $K$ -algebras. We also investigate the image and pre-image of single-valued neutrosophic topological  $K$ -algebras under homomorphism.

**Keywords:**  $K$ -algebras; single-valued neutrosophic sets; homomorphism; compactness;  $C_5$ -connectedness

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## 1. Introduction

A new kind of logical algebra, known as  $K$ -algebra, was introduced by Dar and Akram in [1]. A  $K$ -algebra is built on a group  $G$  by adjoining the induced binary operation on  $G$ . The group  $G$  is particularly of the type in which each non-identity element is not of order 2. This algebraic structure is, in general, non-commutative and non-associative with right identity element [1–3]. Akram et al. [4] introduced fuzzy  $K$ -algebras. They then developed fuzzy  $K$ -algebras with other researchers worldwide. The concepts and results of  $K$ -algebras have been broadened to the fuzzy setting frames by applying Zadeh's fuzzy set theory and its generalizations, namely, interval-valued fuzzy sets, intuitionistic fuzzy sets, interval-valued intuitionistic fuzzy sets, bipolar fuzzy sets and vague sets [5]. In handling information regarding various aspects of uncertainty, non-classical logic is considered to be a more powerful tool than the classical logic. It has become a strong mathematical tool in computer science, medical, engineering, information technology, etc. In 1998, Smarandache [6] introduced neutrosophic set as a generalization of intuitionistic fuzzy set [7]. A neutrosophic set is identified by three functions called truth-membership ( $T$ ), indeterminacy-membership ( $I$ ) and falsity-membership ( $F$ ) functions. To apply neutrosophic set in real-life problems more conveniently, Smarandache [6] and Wang et al. [8] defined single-valued neutrosophic sets which takes the value from the subset of  $[0, 1]$ . Thus, a single-valued neutrosophic set is an instance of neutrosophic set.

Algebraic structures have a vital place with vast applications in various areas of life. Algebraic structures provide a mathematical modeling of related study. Neutrosophic set theory has also been

applied to many algebraic structures. Agboola and Davazz introduced the concept of neutrosophic BCI/BCK-algebras and discuss elementary properties in [9]. Jun et al. introduced the notion of  $(\phi, \psi)$  neutrosophic subalgebra of a BCK/BCI-algebra [10]. Jun et al. [11] defined interval neutrosophic sets on BCK/BCI-algebra [11]. Jun et al. [12] proposed neutrosophic positive implicative  $N$ -ideals and study their extension property [12]. Several set theories and their topological structures have been introduced by many researchers to deal with uncertainties. Chang [13] was the first to introduce the notion of fuzzy topology. Later, Lowan [14], Pu and Liu [15], and Chattopadhyay and Samanta [16] introduced other concepts related to fuzzy topology. Coker [17] introduced the notion of intuitionistic fuzzy topology as a generalization of fuzzy topology. Salama and Alblowi [18] defined the topological structure of neutrosophic set theory. Akram and Dar [19] introduced the concept of fuzzy topological  $K$ -algebras. They extended their work on intuitionistic fuzzy topological  $K$ -algebras [20]. In this paper, we introduce the notion of single-valued neutrosophic topological  $K$ -algebras and investigate some of their properties. Further, we study certain properties, including  $C_5$ -connected, super connected, compact and Hausdorff, of single-valued neutrosophic topological  $K$ -algebras. We also investigate the image and pre-image of single-valued neutrosophic topological  $K$ -algebras under homomorphism.

## 2. Preliminaries

The notion of  $K$ -algebra was introduced by Dar and Akram in [1].

**Definition 1.** [1] Let  $(G, \cdot, e)$  be a group in which each non-identity element is not of order 2. A  $K$ -algebra is a structure  $\mathcal{K} = (G, \cdot, \odot, e)$  over a particular group  $G$ , where  $\odot$  is an induced binary operation  $\odot : G \times G \rightarrow G$  is defined by  $\odot(s, t) = s \odot t = s.t^{-1}$ , and satisfy the following conditions:

- (i)  $(s \odot t) \odot (s \odot u) = (s \odot ((e \odot u) \odot (e \odot t))) \odot s$ ;
- (ii)  $s \odot (s \odot t) = (s \odot (e \odot t)) \odot s$ ;
- (iii)  $s \odot s = e$ ;
- (iv)  $s \odot e = s$ ; and
- (v)  $e \odot s = s^{-1}$

for all  $s, t, u \in G$ . The homomorphism between two  $K$ -algebras  $\mathcal{K}_1$  and  $\mathcal{K}_2$  is a mapping  $f : \mathcal{K}_1 \rightarrow \mathcal{K}_2$  such that, for all  $u, v \in \mathcal{K}_1$ ,  $f(u \odot v) = f(u) \odot f(v)$ .

In [6], Smarandache initiated the idea of neutrosophic set theory which is a generalization of intuitionistic fuzzy set theory. Later, Smarandache and Wang et al. introduced a single-valued neutrosophic set (SNS) as an instance of neutrosophic set in [8].

**Definition 2.** [8] Let  $Z$  be a space of points with a general element  $s \in Z$ . A SNS  $\mathcal{A}$  in  $Z$  is equipped with three membership functions: truth membership function ( $\mathcal{T}_{\mathcal{A}}$ ), indeterminacy membership function ( $\mathcal{I}_{\mathcal{A}}$ ) and falsity membership function ( $\mathcal{F}_{\mathcal{A}}$ ), where  $\forall s \in Z$ ,  $\mathcal{T}_{\mathcal{A}}(s)$ ,  $\mathcal{I}_{\mathcal{A}}(s)$ ,  $\mathcal{F}_{\mathcal{A}}(s) \in [0, 1]$ . There is no restriction on the sum of these three components. Therefore,  $0 \leq \mathcal{T}_{\mathcal{A}}(s) + \mathcal{I}_{\mathcal{A}}(s) + \mathcal{F}_{\mathcal{A}}(s) \leq 3$ .

**Definition 3.** [8] A single-valued neutrosophic empty set ( $\emptyset_{SN}$ ) and single-valued neutrosophic whole set ( $1_{SN}$ ) on  $Z$  is defined as:

- $\emptyset_{SN}(u) = \{u \in Z : (u, 0, 0, 1)\}$ .
- $1_{SN}(u) = \{u \in Z : (u, 1, 1, 0)\}$ .

**Definition 4.** [8] If  $f$  is a mapping from a set  $Z_1$  into a set  $Z_2$ , then the following statements hold:

- (i) Let  $\mathcal{A}$  be a SNS in  $Z_1$  and  $\mathcal{B}$  be a SNS in  $Z_2$ , then the pre-image of  $\mathcal{B}$  is a SNS in  $Z_1$ , denoted by  $f^{-1}(\mathcal{B})$ , defined as:  

$$f^{-1}(\mathcal{B}) = \{z_1 \in Z_1 : f^{-1}(\mathcal{T}_{\mathcal{B}})(z_1) = \mathcal{T}_{\mathcal{B}}(f(z_1)), f^{-1}(\mathcal{I}_{\mathcal{B}})(z_1) = \mathcal{I}_{\mathcal{B}}(f(z_1)), f^{-1}(\mathcal{F}_{\mathcal{B}})(z_1) = \mathcal{F}_{\mathcal{B}}(f(z_1))\}.$$

- (ii) Let  $\mathcal{A} = \{z_1 \in Z_1 : \mathcal{T}_{\mathcal{A}}(z_1), \mathcal{I}_{\mathcal{A}}(z_1), \mathcal{F}_{\mathcal{A}}(z_1)\}$  be a SNS in  $Z_1$  and  $\mathcal{B} = \{z_2 \in Z_2 : \mathcal{T}_{\mathcal{B}}(z_2), \mathcal{I}_{\mathcal{B}}(z_2), \mathcal{F}_{\mathcal{B}}(z_2)\}$  be a SNS in  $Z_2$ . Under the mapping  $f$ , the image of  $\mathcal{A}$  is a SNS in  $Z_2$ , denoted by  $f(\mathcal{A})$ , defined as:  $f(\mathcal{A}) = \{z_2 \in Z_2 : f_{\sup}(\mathcal{T}_{\mathcal{A}})(z_2), f_{\sup}(\mathcal{I}_{\mathcal{A}})(z_2), f_{\inf}(\mathcal{F}_{\mathcal{A}})(z_2)\}$ , where for all  $z_2 \in Z_2$ .

$$f_{\sup}(\mathcal{T}_{\mathcal{A}})(z_2) = \begin{cases} \sup_{z_1 \in f^{-1}(z_2)} \mathcal{T}_{\mathcal{A}}(z_1), & \text{if } f^{-1}(z_2) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f_{\sup}(\mathcal{I}_{\mathcal{A}})(z_2) = \begin{cases} \sup_{z_1 \in f^{-1}(z_2)} \mathcal{I}_{\mathcal{A}}(z_1), & \text{if } f^{-1}(z_2) \neq \emptyset, \\ 0, & \text{otherwise,} \end{cases}$$

$$f_{\inf}(\mathcal{F}_{\mathcal{A}})(z_2) = \begin{cases} \inf_{z_1 \in f^{-1}(z_2)} \mathcal{F}_{\mathcal{A}}(z_1), & \text{if } f^{-1}(z_2) \neq \emptyset, \\ 0, & \text{otherwise.} \end{cases}$$

We formulate the following proposition.

**Proposition 1.** Let  $f : Z_1 \rightarrow Z_2$  and  $\mathcal{A}, (\mathcal{A}_j, j \in J)$  be a SNS in  $Z_1$  and  $\mathcal{B}$  be a SNS in  $Z_2$ . Then,  $f$  possesses the following properties:

- (i) If  $f$  is onto, then  $f(1_{SN}) = 1_{SN}$ .
- (ii)  $f(\emptyset_{SN}) = \emptyset_{SN}$ .
- (iii)  $f^{-1}(1_{SN}) = 1_{SN}$ .
- (iv)  $f^{-1}(\emptyset_{SN}) = \emptyset_{SN}$ .
- (v) If  $f$  is onto, then  $f(f^{-1}(\mathcal{B})) = \mathcal{B}$ .
- (vi)  $f^{-1}(\bigcup_{i=1}^n \mathcal{A}_i) = \bigcup_{i=1}^n f^{-1}(\mathcal{A}_i)$ .

### 3. Neutrosophic Topological K-algebras

**Definition 5.** Let  $Z$  be a nonempty set. A collection  $\chi$  of single-valued neutrosophic sets (SNSs) in  $Z$  is called a single-valued neutrosophic topology (SNT) on  $Z$  if the following conditions hold:

- (a)  $\emptyset_{SN}, 1_{SN} \in \chi$
- (b) If  $\mathcal{A}, \mathcal{B} \in \chi$ , then  $\mathcal{A} \cap \mathcal{B} \in \chi$
- (c) If  $\mathcal{A}_i \in \chi, \forall i \in I$ , then  $\bigcup_{i \in I} \mathcal{A}_i \in \chi$

The pair  $(Z, \chi)$  is called a single-valued neutrosophic topological space (SNTS). Each member of  $\chi$  is said to be  $\chi$ -open or single-valued neutrosophic open set (SNOS) and complement of each open single-valued neutrosophic set is a single-valued neutrosophic closed set (SNCS). A discrete topology is a topology which contains all single-valued neutrosophic subsets of  $Z$  and indiscrete if its elements are only  $\emptyset_{SN}, 1_{SN}$ .

**Definition 6.** Let  $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$  be a single-valued neutrosophic set in  $\mathcal{K}$ . Then,  $\mathcal{A}$  is called a single-valued neutrosophic K-subalgebra of  $\mathcal{K}$  if following conditions hold for  $\mathcal{A}$ :

- (i)  $\mathcal{T}_{\mathcal{A}}(e) \geq \mathcal{T}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(e) \geq \mathcal{I}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(e) \leq \mathcal{F}_{\mathcal{A}}(s)$ .
- (ii)  $\mathcal{T}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{T}_{\mathcal{A}}(s), \mathcal{T}_{\mathcal{A}}(t)\},$   
 $\mathcal{I}_{\mathcal{A}}(s \odot t) \geq \min\{\mathcal{I}_{\mathcal{A}}(s), \mathcal{I}_{\mathcal{A}}(t)\},$   
 $\mathcal{F}_{\mathcal{A}}(s \odot t) \leq \max\{\mathcal{F}_{\mathcal{A}}(s), \mathcal{F}_{\mathcal{A}}(t)\} \forall s, t \in \mathcal{K}.$

**Example 1.** Consider a  $K$ -algebra  $\mathcal{K} = (G, \cdot, \odot, e)$ , where  $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$  is the cyclic group of order 9 and Caley's table for  $\odot$  is given as:

$\odot$	$e$	$x$	$x^2$	$x^3$	$x^4$	$x^5$	$x^6$	$x^7$	$x^8$
$e$	$e$	$x^8$	$x^7$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$	$x$
$x$	$x$	$e$	$x^8$	$x^7$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$
$x^2$	$x^2$	$x$	$e$	$x^8$	$x^7$	$x^6$	$x^5$	$x^4$	$x^3$
$x^3$	$x^3$	$x^2$	$x$	$e$	$x^8$	$x^7$	$x^6$	$x^5$	$x^4$
$x^4$	$x^4$	$x^3$	$x^2$	$x$	$e$	$x^8$	$x^7$	$x^6$	$x^5$
$x^5$	$x^5$	$x^4$	$x^3$	$x^2$	$x$	$e$	$x^8$	$x^7$	$x^6$
$x^6$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$	$x$	$e$	$x^8$	$x^7$
$x^7$	$x^7$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$	$x$	$e$	$x^8$
$x^8$	$x^8$	$x^7$	$x^6$	$x^5$	$x^4$	$x^3$	$x^2$	$x$	$e$

If we define a single-valued neutrosophic set  $\mathcal{A}, \mathcal{B}$  in  $\mathcal{K}$  such that:

$$\mathcal{A} = \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.7)\},$$

$$\mathcal{B} = \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\}$$

$\forall s \neq e \in G$ .

According to Definition 5, the family  $\{\mathcal{O}_{\text{SN}}, 1_{\text{SN}}, \mathcal{A}, \mathcal{B}\}$  of SNSs of  $K$ -algebra is a SNT on  $\mathcal{K}$ . We define a SNS  $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$  in  $\mathcal{K}$  such that  $\mathcal{T}_{\mathcal{A}}(e) = 0.7, \mathcal{I}_{\mathcal{A}}(e) = 0.5, \mathcal{F}_{\mathcal{A}}(e) = 0.2, \mathcal{T}_{\mathcal{A}}(s) = 0.2, \mathcal{I}_{\mathcal{A}}(s) = 0.4, \mathcal{F}_{\mathcal{A}}(s) = 0.6$ . Clearly,  $\mathcal{A} = (\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}})$  is a SN  $K$ -subalgebra of  $\mathcal{K}$ .

**Definition 7.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a  $K$ -algebra and let  $\chi_{\mathcal{K}}$  be a topology on  $\mathcal{K}$ . Let  $\mathcal{A}$  be a SNS in  $\mathcal{K}$  and let  $\chi_{\mathcal{K}}$  be a topology on  $\mathcal{K}$ . Then, an induced single-valued neutrosophic topology on  $\mathcal{A}$  is a collection or family of single-valued neutrosophic subsets of  $\mathcal{A}$  which are the intersection with  $\mathcal{A}$  and single-valued neutrosophic open sets in  $\mathcal{K}$  defined as  $\chi_{\mathcal{A}} = \{\mathcal{A} \cap F : F \in \chi_{\mathcal{K}}\}$ . Then,  $\chi_{\mathcal{A}}$  is called single-valued neutrosophic induced topology on  $\mathcal{A}$  or relative topology and the pair  $(\mathcal{A}, \chi_{\mathcal{A}})$  is called an induced topological space or single-valued neutrosophic subspace of  $(\mathcal{K}, \chi_{\mathcal{K}})$ .

**Definition 8.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTs and let  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$ . Then,  $f$  is called single-valued neutrosophic continuous if following conditions hold:

- (i) For each SNS  $\mathcal{A} \in \chi_2, f^{-1}(\mathcal{A}) \in \chi_1$ .
- (ii) For each SN  $K$ -subalgebra  $\mathcal{A} \in \chi_2, f^{-1}(\mathcal{A})$  is a SN  $K$ -subalgebra  $\in \chi_1$ .

**Definition 9.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTs and let  $(\mathcal{A}, \chi_{\mathcal{A}})$  and  $(\mathcal{B}, \chi_{\mathcal{B}})$  be two single-valued neutrosophic subspaces over  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$ . Let  $f$  be a mapping from  $(\mathcal{K}_1, \chi_1)$  into  $(\mathcal{K}_2, \chi_2)$ , then  $f$  is a mapping from  $(\mathcal{A}, \chi_{\mathcal{A}})$  to  $(\mathcal{B}, \chi_{\mathcal{B}})$  if  $f(\mathcal{A}) \subset \mathcal{B}$ .

**Definition 10.** Let  $f$  be a mapping from  $(\mathcal{A}, \chi_{\mathcal{A}})$  to  $(\mathcal{B}, \chi_{\mathcal{B}})$ . Then,  $f$  is relatively single-valued neutrosophic continuous if for every SNOS  $Y_{\mathcal{B}}$  in  $\chi_{\mathcal{B}}, f^{-1}(Y_{\mathcal{B}}) \cap \mathcal{A} \in \chi_{\mathcal{A}}$ .

**Definition 11.** Let  $f$  be a mapping from  $(\mathcal{A}, \chi_{\mathcal{A}})$  to  $(\mathcal{B}, \chi_{\mathcal{B}})$ . Then,  $f$  is relatively single-valued neutrosophic open if for every SNOS  $X_{\mathcal{A}}$  in  $\chi_{\mathcal{A}},$  the image  $f(X_{\mathcal{A}}) \in \chi_{\mathcal{B}}$ .

**Proposition 2.** Let  $(\mathcal{A}, \chi_{\mathcal{A}})$  and  $(\mathcal{B}, \chi_{\mathcal{B}})$  be single-valued neutrosophic subspaces of  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$ , where  $\mathcal{K}_1$  and  $\mathcal{K}_2$  are  $K$ -algebras. If  $f$  is a single-valued neutrosophic continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$  and  $f(\mathcal{A}) \subset \mathcal{B}$ . Then,  $f$  is relatively single-valued neutrosophic continuous function from  $\mathcal{A}$  into  $\mathcal{B}$ .

**Definition 12.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTs. A mapping  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  is called a single-valued neutrosophic homomorphism if following conditions hold:

- (i)  $f$  is a one-one and onto function.
- (ii)  $f$  is a single-valued neutrosophic continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$ .
- (iii)  $f^{-1}$  is a single-valued neutrosophic continuous function from  $\mathcal{K}_2$  to  $\mathcal{K}_1$ .

**Theorem 1.** Let  $(\mathcal{K}_1, \chi_1)$  be a SNTS and  $(\mathcal{K}_2, \chi_2)$  be an indiscrete SNTS on  $K$ -algebras  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively. Then, each function  $f$  defined as  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  is a single-valued neutrosophic continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$ . If  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two discrete SNTSs  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively, then each homomorphism  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  is a single values neutrosophic continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$ .

**Proof.** Let  $f$  be a mapping defined as  $f : \mathcal{K}_1 \rightarrow \mathcal{K}_2$ . Let  $\chi_1$  be SNT on  $\mathcal{K}_1$  and  $\chi_2$  be SNT on  $\mathcal{K}_2$ , where  $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$ . We show that  $f^{-1}(\mathcal{A})$  is a single-valued neutrosophic  $K$ -subalgebra of  $\mathcal{K}_1$ , i.e., for each  $\mathcal{A} \in \chi_2$ ,  $f^{-1}(\mathcal{A}) \in \chi_1$ . Since  $\chi_2 = \{\emptyset_{SN}, 1_{SN}\}$ , then for any  $u \in \chi_1$ , consider  $\emptyset_{SN} \in \chi_2$  such that  $f^{-1}(\emptyset_{SN})(u) = \emptyset_{SN}(f(u)) = \emptyset_{SN}(u)$ .

Therefore,  $(f^{-1}(\emptyset_{SN})) = \emptyset_{SN} \in \chi_1$ . Likewise,  $(f^{-1}(1_{SN})) = 1_{SN} \in \chi_1$ . Hence,  $f$  is a SN continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$ .

Now, for the second part of the theorem, where both  $\chi_1$  and  $\chi_2$  are SNTSs on  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively, and  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  is a homomorphism. Therefore, for all  $\mathcal{A} \in \chi_2$  and  $f^{-1}\mathcal{A} \in \chi_1$ , where  $f$  is not a usual inverse homomorphism. To prove that  $f^{-1}(\mathcal{A})$  is a single-valued neutrosophic  $K$ -subalgebra in of  $\mathcal{K}_1$ . Let for  $u, v \in \mathcal{K}_1$ ,

$$\begin{aligned} f^{-1}(\mathcal{T}_A)(u \odot v) &= \mathcal{T}_A(f(u \odot v)) \\ &= \mathcal{T}_A(f(u) \odot f(v)) \\ &\geq \min\{\mathcal{T}_A(f(u)) \odot \mathcal{T}_A(f(v))\} \\ &= \min\{f^{-1}(\mathcal{T}_A)(u), f^{-1}(\mathcal{T}_A)(v)\}, \\ f^{-1}(\mathcal{I}_A)(u \odot v) &= \mathcal{I}_A(f(u \odot v)) \\ &= \mathcal{I}_A(f(u) \odot f(v)) \\ &\geq \min\{\mathcal{I}_A(f(u)) \odot \mathcal{I}_A(f(v))\} \\ &= \min\{f^{-1}(\mathcal{I}_A)(u), f^{-1}(\mathcal{I}_A)(v)\}, \\ f^{-1}(\mathcal{F}_A)(u \odot v) &= \mathcal{F}_A(f(u \odot v)) \\ &= \mathcal{F}_A(f(u) \odot f(v)) \\ &\leq \max\{\mathcal{F}_A(f(u)) \odot \mathcal{F}_A(f(v))\} \\ &= \max\{f^{-1}(\mathcal{F}_A)(u), f^{-1}(\mathcal{F}_A)(v)\}. \end{aligned}$$

Hence,  $f$  is a single-valued neutrosophic continuous function from  $\mathcal{K}_1$  to  $\mathcal{K}_2$ .  $\square$

**Proposition 3.** Let  $\chi_1$  and  $\chi_2$  be two SNTSs on  $\mathcal{K}$ . Then, each homomorphism  $f : (\mathcal{K}, \chi_1) \rightarrow (\mathcal{K}, \chi_2)$  is a single-valued neutrosophic continuous function.

**Proof.** Let  $(\mathcal{K}, \chi_1)$  and  $(\mathcal{K}, \chi_2)$  be two SNTSs, where  $\mathcal{K}$  is a  $K$ -algebra. To prove the above result, it is enough to show that result is false for a particular topology. Let  $\mathcal{A} = (\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A)$  and  $\mathcal{B} = (\mathcal{T}_B, \mathcal{I}_B, \mathcal{F}_B)$  be two SNSs in  $\mathcal{K}$ . Take  $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$  and  $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{B}\}$ . If  $f : (\mathcal{K}, \chi_1) \rightarrow (\mathcal{K}, \chi_2)$ , defined by  $f(u) = e \odot u$ , for all  $u \in \mathcal{K}$ , then  $f$  is a homomorphism. Now, for  $u \in \mathcal{A}$ ,  $v \in \chi_2$ ,  $(f^{-1}(\mathcal{B}))(u) = \mathcal{B}(f(u)) = \mathcal{B}(e \odot u) = \mathcal{B}(u)$ ,  $\forall u \in \mathcal{K}$ , i.e.,  $f^{-1}(\mathcal{B}) = \mathcal{B}$ . Therefore,  $(f^{-1}(\mathcal{B})) \notin \chi_1$ . Hence,  $f$  is not a single-valued neutrosophic continuous mapping.  $\square$

**Definition 13.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a  $K$ -algebra and  $\chi$  be a SNT on  $\mathcal{K}$ . Let  $\mathcal{A}$  be a single-valued neutrosophic  $K$ -algebra ( $K$ -subalgebra) of  $\mathcal{K}$  and  $\chi_A$  be a SNT on  $\mathcal{A}$ . Then,  $\mathcal{A}$  is said to be a single-valued neutrosophic topological  $K$ -algebra ( $K$ -subalgebra) on  $\mathcal{K}$  if the self mapping  $\rho_a : (\mathcal{A}, \chi_A) \rightarrow (\mathcal{A}, \chi_A)$  defined as  $\rho_a(u) = u \odot a, \forall a \in \mathcal{K}$ , is a relatively single-valued neutrosophic continuous mapping.

**Theorem 2.** Let  $\chi_1$  and  $\chi_2$  be two SNTSs on  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively, and  $f : \mathcal{K}_1 \rightarrow \mathcal{K}_2$  be a homomorphism such that  $f^{-1}(\chi_2) = \chi_1$ . If  $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$  is a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_2$ , then  $f^{-1}(\mathcal{A})$  is a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_1$ .

**Proof.** Let  $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$  be a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_2$ . To prove that  $f^{-1}(\mathcal{A})$  be a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_1$ . Let for any  $u, v \in \mathcal{K}_1$ ,

$$\begin{aligned}\mathcal{T}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{T}_{\mathcal{A}}(f(u \odot v)) \\ &\geq \min\{\mathcal{T}_{\mathcal{A}}(f(u)), \mathcal{T}_{\mathcal{A}}(f(v))\} \\ &= \min\{\mathcal{T}_{f^{-1}(\mathcal{A})}(u), \mathcal{T}_{f^{-1}(\mathcal{A})}(v)\}, \\ \mathcal{I}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{I}_{\mathcal{A}}(f(u \odot v)) \\ &\geq \min\{\mathcal{I}_{\mathcal{A}}(f(u)), \mathcal{I}_{\mathcal{A}}(f(v))\} \\ &= \min\{\mathcal{I}_{f^{-1}(\mathcal{A})}(u), \mathcal{I}_{f^{-1}(\mathcal{A})}(v)\}, \\ \mathcal{F}_{f^{-1}(\mathcal{A})}(u \odot v) &= \mathcal{F}_{\mathcal{A}}(f(u \odot v)) \\ &\leq \max\{\mathcal{F}_{\mathcal{A}}(f(u)), \mathcal{F}_{\mathcal{A}}(f(v))\} \\ &= \max\{\mathcal{F}_{f^{-1}(\mathcal{A})}(u), \mathcal{F}_{f^{-1}(\mathcal{A})}(v)\}.\end{aligned}$$

Hence,  $f^{-1}(\mathcal{A})$  is a single-valued neutrosophic K-algebra of  $\mathcal{K}_1$ .

Now, we prove that  $f^{-1}(\mathcal{A})$  is single-valued neutrosophic topological K-algebra of  $\mathcal{K}_1$ . Since  $f$  is a single-valued neutrosophic continuous function, then by proposition 3.1,  $f$  is also a relatively single-valued neutrosophic continuous function which maps  $(f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$  to  $(\mathcal{A}, \chi_{\mathcal{A}})$ .

Let  $a \in \mathcal{K}_1$  and  $Y$  be a SNS in  $\chi_{\mathcal{A}}$ , and let  $X$  be a SNS in  $\chi_{f^{-1}(\mathcal{A})}$  such that

$$f^{-1}(Y) = X. \quad (1)$$

We are to prove that  $\rho_a : (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})}) \rightarrow (f^{-1}(\mathcal{A}), \chi_{f^{-1}(\mathcal{A})})$  is relatively single-valued neutrosophic continuous mapping, then for any  $a \in \mathcal{K}_1$ , we have

$$\begin{aligned}\mathcal{T}_{\rho_a^{-1}(X)}(u) &= \mathcal{T}_{(X)}(\rho_a(u)) = \mathcal{T}_{(X)}(u \odot a) \\ &= \mathcal{T}_{f^{-1}(Y)}(u \odot a) = \mathcal{T}_{(Y)}(f(u \odot a)) \\ &= \mathcal{T}_{(Y)}(f(u) \odot f(a)) = \mathcal{T}_{(Y)}(\rho_{f(a)}(f(u))) \\ &= \mathcal{T}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{T}_{f^{-1}(\rho_{f(a)}^{-1}(Y))}(u), \\ \mathcal{I}_{\rho_a^{-1}(X)}(u) &= \mathcal{I}_{(X)}(\rho_a(u)) = \mathcal{I}_{(X)}(u \odot a) \\ &= \mathcal{I}_{f^{-1}(Y)}(u \odot a) = \mathcal{I}_{(Y)}(f(u \odot a)) \\ &= \mathcal{I}_{(Y)}(f(u) \odot f(a)) = \mathcal{I}_{(Y)}(\rho_{f(a)}(f(u))) \\ &= \mathcal{I}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{I}_{f^{-1}(\rho_{f(a)}^{-1}(Y))}(u), \\ \mathcal{F}_{\rho_a^{-1}(X)}(u) &= \mathcal{F}_{(X)}(\rho_a(u)) = \mathcal{F}_{(X)}(u \odot a) \\ &= \mathcal{F}_{f^{-1}(Y)}(u \odot a) = \mathcal{F}_{(Y)}(f(u \odot a)) \\ &= \mathcal{F}_{(Y)}(f(u) \odot f(a)) = \mathcal{F}_{(Y)}(\rho_{f(a)}(f(u))) \\ &= \mathcal{F}_{\rho^{-1}f(a)Y}(f(u)) = \mathcal{F}_{f^{-1}(\rho_{f(a)}^{-1}(Y))}(u).\end{aligned}$$

It concludes that  $\rho_a^{-1}(X) = f^{-1}(\rho_{f(a)}^{-1}(Y))$ . Thus,  $\rho_a^{-1}(X) \cap f^{-1}(\mathcal{A}) = f^{-1}(\rho_{f(a)}^{-1}(Y)) \cap f^{-1}(\mathcal{A})$  is a SNS in  $f^{-1}(\mathcal{A})$  and a SNS in  $\chi_{f^{-1}(\mathcal{A})}$ . Hence,  $f^{-1}(\mathcal{A})$  and a single-valued neutrosophic topological K-algebra of  $\mathcal{K}$ . Hence, the proof.  $\square$

**Theorem 3.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTSs on  $\mathcal{K}_1$  and  $\mathcal{K}_2$ , respectively, and let  $f$  be a bijective homomorphism of  $\mathcal{K}_1$  into  $\mathcal{K}_2$  such that  $f(\chi_1) = \chi_2$ . If  $\mathcal{A}$  is a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_1$ , then  $f(\mathcal{A})$  is a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_2$ .

**Proof.** Suppose that  $\mathcal{A} = \{\mathcal{T}_{\mathcal{A}}, \mathcal{I}_{\mathcal{A}}, \mathcal{F}_{\mathcal{A}}\}$  is a SN topological K-algebra of  $\mathcal{K}_1$ . To prove that  $f(\mathcal{A})$  is a single-valued neutrosophic topological K-algebra of  $\mathcal{K}_2$ , let, for  $u, v \in \mathcal{K}_2$ ,

$$f(\mathcal{A}) = (f_{\sup}(\mathcal{T}_{\mathcal{A}})(v), f_{\sup}(\mathcal{I}_{\mathcal{A}})(v), f_{\inf}(\mathcal{F}_{\mathcal{A}})(v)).$$

Let  $a_o \in f^{-1}(u)$ ,  $b_o \in f^{-1}(v)$  such that

$$\begin{aligned} \sup_{x \in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x) &= \mathcal{T}_{\mathcal{A}}(a_o), \sup_{x \in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x) = \mathcal{T}_{\mathcal{A}}(b_o), \\ \sup_{x \in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x) &= \mathcal{I}_{\mathcal{A}}(a_o), \sup_{x \in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x) = \mathcal{I}_{\mathcal{A}}(b_o), \\ \inf_{x \in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x) &= \mathcal{F}_{\mathcal{A}}(a_o), \inf_{x \in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x) = \mathcal{F}_{\mathcal{A}}(b_o). \end{aligned}$$

Now,

$$\begin{aligned} \mathcal{T}_{f(\mathcal{A})}(u \odot v) &= \sup_{x \in f^{-1}(u \odot v)} \mathcal{T}_{\mathcal{A}}(x) \\ &\geq \mathcal{T}_{\mathcal{A}}(a_o, b_o) \\ &\geq \min\{\mathcal{T}_{\mathcal{A}}(a_o), \mathcal{T}_{\mathcal{A}}(b_o)\} \\ &= \min\left\{\sup_{x \in f^{-1}(u)} \mathcal{T}_{\mathcal{A}}(x), \sup_{x \in f^{-1}(v)} \mathcal{T}_{\mathcal{A}}(x)\right\} \\ &= \min\{\mathcal{T}_{f(\mathcal{A})}(u), \mathcal{T}_{f(\mathcal{A})}(v)\}, \end{aligned}$$

$$\begin{aligned} \mathcal{I}_{f(\mathcal{A})}(u \odot v) &= \sup_{x \in f^{-1}(u \odot v)} \mathcal{I}_{\mathcal{A}}(x) \\ &\geq \mathcal{I}_{\mathcal{A}}(a_o, b_o) \\ &\geq \min\{\mathcal{I}_{\mathcal{A}}(a_o), \mathcal{I}_{\mathcal{A}}(b_o)\} \\ &= \min\left\{\sup_{x \in f^{-1}(u)} \mathcal{I}_{\mathcal{A}}(x), \sup_{x \in f^{-1}(v)} \mathcal{I}_{\mathcal{A}}(x)\right\} \\ &= \min\{\mathcal{I}_{f(\mathcal{A})}(u), \mathcal{I}_{f(\mathcal{A})}(v)\}, \end{aligned}$$

$$\begin{aligned} \mathcal{F}_{f(\mathcal{A})}(u \odot v) &= \inf_{x \in f^{-1}(u \odot v)} \mathcal{F}_{\mathcal{A}}(x) \\ &\leq \mathcal{F}_{\mathcal{A}}(a_o, b_o) \\ &\leq \max\{\mathcal{F}_{\mathcal{A}}(a_o), \mathcal{F}_{\mathcal{A}}(b_o)\} \\ &= \max\left\{\inf_{x \in f^{-1}(u)} \mathcal{F}_{\mathcal{A}}(x), \inf_{x \in f^{-1}(v)} \mathcal{F}_{\mathcal{A}}(x)\right\} \\ &= \max\{\mathcal{F}_{f(\mathcal{A})}(u), \mathcal{F}_{f(\mathcal{A})}(v)\}. \end{aligned}$$

Hence,  $f(\mathcal{A})$  is a single-valued neutrosophic  $K$ -subalgebra of  $\mathcal{K}_2$ . Now, we prove that the self mapping  $\rho_b : (f(\mathcal{A}), \chi_f(\mathcal{A})) \rightarrow (f(\mathcal{A}), \chi_f(\mathcal{A}))$ , defined by  $\rho_b(v) = v \odot b$ , for all  $b \in \mathcal{K}_2$ , is a relatively single-valued neutrosophic continuous mapping. Let  $Y_{\mathcal{A}}$  be a SNS in  $\chi_{\mathcal{A}}$ , there exists a SNS “ $Y$ ” in  $\chi_1$  such that  $Y_{\mathcal{A}} = Y \cap \mathcal{A}$ . We show that for a SNS in  $\chi_{f(\mathcal{A})}$ ,

$$\rho^{-1}_b(Y_{f(\mathcal{A})}) \cap f(\mathcal{A}) \in \chi_{f(\mathcal{A})}$$

Since  $f$  is an injective mapping, then  $f(Y_{\mathcal{A}}) = f(Y \cap \mathcal{A}) = f(Y) \cap f(\mathcal{A})$  is a SNS in  $\chi_{f(\mathcal{A})}$  which shows that  $f$  is relatively single-valued neutrosophic open. In addition,  $f$  is surjective, then for all  $b \in \mathcal{K}_2$ ,  $a = f(b)$ , where  $a \in \mathcal{K}_1$ .

Now,

$$\begin{aligned}
 \mathcal{T}_{f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}})))(u)} &= \mathcal{T}_{f^{-1}(\rho^{-1}_f(a)(Y_{f(\mathcal{A}})))(u)} \\
 &= \mathcal{T}_{\rho^{-1}_f(a)(Y_{f(\mathcal{A}})}(f(u)) \\
 &= \mathcal{T}_{(Y_{f(\mathcal{A}})}(\rho_{f(a)}(f(u))) \\
 &= \mathcal{T}_{(Y_{f(\mathcal{A}})}(f(u) \odot f(a)) \\
 &= \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A}})}(u \odot a)) \\
 &= \mathcal{T}_{f^{-1}(Y_{f(\mathcal{A}})}(\rho_a(u)) \\
 &= \mathcal{T}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A}})))(u),
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{I}_{f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}})))(u)} &= \mathcal{I}_{f^{-1}(\rho^{-1}_f(a)(Y_{f(\mathcal{A}})))(u)} \\
 &= \mathcal{I}_{\rho^{-1}_f(a)(Y_{f(\mathcal{A}})}(f(u)) \\
 &= \mathcal{I}_{(Y_{f(\mathcal{A}})}(\rho_{f(a)}(f(u))) \\
 &= \mathcal{I}_{(Y_{f(\mathcal{A}})}(f(u) \odot f(a)) \\
 &= \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A}})}(u \odot a)) \\
 &= \mathcal{I}_{f^{-1}(Y_{f(\mathcal{A}})}(\rho_a(u)) \\
 &= \mathcal{I}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A}})))(u),
 \end{aligned}$$

$$\begin{aligned}
 \mathcal{F}_{f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}})))(u)} &= \mathcal{F}_{f^{-1}(\rho^{-1}_f(a)(Y_{f(\mathcal{A}})))(u)} \\
 &= \mathcal{F}_{\rho^{-1}_f(a)(Y_{f(\mathcal{A}})}(f(u)) \\
 &= \mathcal{F}_{(Y_{f(\mathcal{A}})}(\rho_{f(a)}(f(u))) \\
 &= \mathcal{F}_{(Y_{f(\mathcal{A}})}(f(u) \odot f(a)) \\
 &= \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A}})}(u \odot a)) \\
 &= \mathcal{F}_{f^{-1}(Y_{f(\mathcal{A}})}(\rho_a(u)) \\
 &= \mathcal{F}_{\rho^{-1}(a)}(f^{-1}(Y_{f(\mathcal{A}})))(u).
 \end{aligned}$$

This implies that  $f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}}))) = \rho^{-1}_a(f^{-1}(Y_{f(\mathcal{A}})))$ . Since  $\rho_a : (\mathcal{A}, \chi_{\mathcal{A}}) \rightarrow (\mathcal{A}, \chi_{\mathcal{A}})$  is relatively single-valued neutrosophic continuous mapping and  $f$  is relatively single-valued neutrosophic continues mapping from  $(\mathcal{A}, \chi_{\mathcal{A}})$  into  $(f(\mathcal{A}), \chi_{f(\mathcal{A})})$ ,  $f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}}))) \cap \mathcal{A} = \rho^{-1}_a(f^{-1}(Y_{f(\mathcal{A}}))) \cap \mathcal{A}$  is a SNS in  $\chi_{\mathcal{A}}$ . Hence,  $f(f^{-1}(\rho^{-1}_b(Y_{f(\mathcal{A}}))) \cap \mathcal{A}) = \rho^{-1}_b(Y_{f(\mathcal{A}})) \cap f(\mathcal{A})$  is a SNS in  $\chi_{\mathcal{A}}$ , which completes the proof.  $\square$

**Example 2.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a  $K$ -algebra, where  $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$  is the cyclic group of order 9 and Caley's table for  $\odot$  is given in Example 1. We define a SNS as:

$$\begin{aligned}
 \mathcal{A} &= \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\}, \\
 \mathcal{B} &= \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\},
 \end{aligned}$$

for all  $s \neq e \in G$ , where  $\mathcal{A}, \mathcal{B} \in [0, 1]$ . The collection  $\chi_{\mathcal{K}} = \{\emptyset_{\text{SN}}, 1_{\text{SN}}, \mathcal{A}, \mathcal{B}\}$  of SNSs of  $\mathcal{K}$  is a SNT on  $\mathcal{K}$  and  $(\mathcal{K}, \chi_{\mathcal{K}})$  is a SNTS. Let  $\mathcal{C}$  be a SNS in  $\mathcal{K}$ , defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \forall s \neq e \in G.$$



Clearly,  $\mathcal{C}$  is a single-valued neutrosophic  $K$ -subalgebra of  $\mathcal{K}$ . By direct calculations relative topology  $\chi_{\mathcal{C}}$  is obtained as  $\chi_{\mathcal{C}} = \{\emptyset_{\mathcal{A}}, 1_{\mathcal{A}}, \mathcal{A}\}$ . Then, the pair  $(\mathcal{C}, \chi_{\mathcal{C}})$  is a single-valued neutrosophic subspace of  $(\mathcal{K}, \chi_{\mathcal{K}})$ . We show that  $\mathcal{C}$  is a single-valued neutrosophic topological  $K$ -subalgebra of  $\mathcal{K}$ , i.e., the self mapping  $\rho_a : (\mathcal{C}, \chi_{\mathcal{C}}) \rightarrow (\mathcal{C}, \chi_{\mathcal{C}})$  defined by  $\rho_a(u) = u \odot a, \forall a \in \mathcal{K}$  is relatively single-valued neutrosophic continuous mapping, i.e., for a SNOS  $\mathcal{A}$  in  $(\mathcal{C}, \chi_{\mathcal{C}})$ ,  $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} \in \chi_{\mathcal{C}}$ . Since  $\rho_a$  is homomorphism, then  $\rho_a^{-1}(\mathcal{A}) \cap \mathcal{C} = \mathcal{A} \in \chi_{\mathcal{C}}$ . Therefore,  $\rho_a : (\mathcal{C}, \chi_{\mathcal{C}}) \rightarrow (\mathcal{C}, \chi_{\mathcal{C}})$  is relatively single-valued neutrosophic continuous mapping. Hence,  $\mathcal{C}$  is a single-valued neutrosophic topological  $K$ -algebra of  $\mathcal{K}$ .

**Example 3.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a  $K$ -algebra, where  $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$  is the cyclic group of order 9 and Caley's table for  $\odot$  is given in Example 3.1. We define a SNS as:

$$\begin{aligned}\mathcal{A} &= \{(e, 0.4, 0.5, 0.8), (s, 0.3, 0.4, 0.6)\}, \\ \mathcal{B} &= \{(e, 0.3, 0.4, 0.8), (s, 0.2, 0.3, 0.6)\}, \\ \mathcal{D} &= \{(e, 0.2, 0.1, 0.3), (s, 0.1, 0.1, 0.5)\},\end{aligned}$$

for all  $s \neq e \in G$ , where  $\mathcal{A}, \mathcal{B} \in [0, 1]$ . The collection  $\chi_1 = \{\emptyset_{SN}, 1_{SN}, \mathcal{D}\}$  and  $\chi_2 = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}, \mathcal{B}\}$  of SNSs of  $\mathcal{K}$  are SNTs on  $\mathcal{K}$  and  $(\mathcal{K}, \chi_1), (\mathcal{K}, \chi_2)$  be two SNTSs. Let  $\mathcal{C}$  be a SNS in  $(\mathcal{K}, \chi_2)$ , defined as:

$$\mathcal{C} = \{(e, 0.7, 0.5, 0.2), (s, 0.5, 0.4, 0.6)\}, \forall s \neq e \in G.$$

Now, Let  $f : (\mathcal{K}, \chi_1) \rightarrow (\mathcal{K}, \chi_2)$  be a homomorphism such that  $f^{-1}(\chi_2) = \chi_1$  (we have not consider  $\mathcal{K}$  to be distinct), then, by Proposition 3,  $f$  is a single-valued neutrosophic continuous function and  $f$  is also relatively single-valued neutrosophic continues mapping from  $(\mathcal{K}, \chi_1)$  into  $(\mathcal{K}, \chi_2)$ . Since  $\mathcal{C}$  is a SNS in  $(\mathcal{K}, \chi_2)$  and with relative topology  $\chi_{\mathcal{C}} = \{\emptyset_{\mathcal{A}}, 1_{\mathcal{A}}, \mathcal{A}\}$  is also a single-valued neutrosophic topological  $K$ -algebra of  $(\mathcal{K}, \chi_2)$ . We prove that  $f^{-1}(\mathcal{C})$  is a single-valued neutrosophic topological  $K$ -algebra in  $(\mathcal{K}, \chi_1)$ . Since  $f$  is a continuous function, then, by Definition 8,  $f^{-1}(\mathcal{C})$  is a single-valued neutrosophic  $K$ -subalgebra in  $(\mathcal{K}, \chi_1)$ . To prove that  $f^{-1}(\mathcal{C})$  is a single-valued neutrosophic topological  $K$ -algebra, then for  $b \in \mathcal{K}_1$  take

$$\rho_b : (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}) \rightarrow (f^{-1}(\mathcal{C}), \chi_{f^{-1}(\mathcal{C})}),$$

for  $\mathcal{A} \in \chi_{f^{-1}(\mathcal{C})}$ ,  $\rho_b^{-1}(\mathcal{A}) \cap f^{-1}(\mathcal{C}) \in \chi_{f^{-1}(\mathcal{C})}$  which shows that  $f^{-1}(\mathcal{C})$  is a single-valued neutrosophic topological  $K$ -algebra in  $(\mathcal{K}, \chi_1)$ . Similarly, we can show that  $f(\mathcal{C})$  is a single-valued neutrosophic topological  $K$ -algebra in  $(\mathcal{K}, \chi_2)$  by considering a bijective homomorphism.

**Definition 14.** Let  $\chi$  be a SNT on  $\mathcal{K}$  and  $(\mathcal{K}, \chi)$  be a SNTS. Then,  $(\mathcal{K}, \chi)$  is called single-valued neutrosophic  $C_5$ -disconnected topological space if there exist a SNOS and SNCS  $\mathcal{H}$  such that  $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}}) \neq 1_{SN}$  and  $\mathcal{H} = (\mathcal{T}_{\mathcal{H}}, \mathcal{I}_{\mathcal{H}}, \mathcal{F}_{\mathcal{H}}) \neq \emptyset_{SN}$ , otherwise  $(\mathcal{K}, \chi)$  is called single-valued neutrosophic  $C_5$ -connected.

**Example 4.** Every indiscrete SNT space on  $\mathcal{K}$  is  $C_5$ -connected.

**Proposition 4.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTSs and  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  be a surjective single-valued neutrosophic continuous mapping. If  $(\mathcal{K}_1, \chi_1)$  is a single-valued neutrosophic  $C_5$ -connected space, then  $(\mathcal{K}_2, \chi_2)$  is also a single-valued neutrosophic  $C_5$ -connected space.

**Proof.** Suppose on contrary that  $(\mathcal{K}_2, \chi_2)$  is a single-valued neutrosophic  $C_5$ -disconnected space. Then, by Definition 14, there exist both SNOS and SNCS  $\mathcal{H}$  be such that  $\mathcal{H} \neq 1_{SN}$  and  $\mathcal{H} \neq \emptyset_{SN}$ . Since  $f$  is a single-valued neutrosophic continuous and onto function, so  $f^{-1}(\mathcal{H}) = 1_{SN}$  or  $f^{-1}(\mathcal{H}) = \emptyset_{SN}$ , where  $f^{-1}(\mathcal{H})$  is both SNOS and SNCS. Therefore,

$$\mathcal{H} = f(f^{-1}(\mathcal{H})) = f(1_{SN}) = 1_{SN} \quad (2)$$

and

$$\mathcal{H} = f(f^{-1}(\mathcal{H})) = f(\emptyset_{SN}) = \emptyset_{SN}, \quad (3)$$

a contradiction. Hence,  $(\mathcal{K}_2, \chi_2)$  is a single-valued neutrosophic  $C_5$ -connected space.  $\square$

**Corollary 1.** Let  $\chi$  be a SNT on  $\mathcal{K}$ . Then,  $(\mathcal{K}, \chi)$  is called a single-valued neutrosophic  $C_5$ -connected space if and only if there does not exist a single-valued neutrosophic continuous map  $f : (\mathcal{K}, \chi) \rightarrow (\mathcal{F}_T, \chi_T)$  such that  $f \neq 1_{SN}$  and  $f \neq \emptyset_{SN}$ .

**Definition 15.** Let  $\mathcal{A} = \{\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A\}$  be a SNS in  $\mathcal{K}$ . Let  $\chi$  be a SNT on  $\mathcal{K}$ . The interior and closure of  $\mathcal{A}$  in  $\mathcal{K}$  is defined as:

$\mathcal{A}^{Int}$ : The union of SNOSs which contained in  $\mathcal{A}$ .

$\mathcal{A}^{Clo}$ : The intersection of SNCSs for which  $\mathcal{A}$  is a subset of these SNCSs.

**Remark 1.** Being union of SNOS  $\mathcal{A}^{Int}$  is a SNO and  $\mathcal{A}^{Clo}$  being intersection of SNCS is SNC.

**Theorem 4.** Let  $\mathcal{A}$  be a SNS in a SNTS  $(\mathcal{K}, \chi)$ . Then,  $\mathcal{A}^{Int}$  is such an open set which is the largest open set of  $\mathcal{K}$  contained in  $\mathcal{A}$ .

**Corollary 2.**  $\mathcal{A} = (\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A)$  is a SNOS in  $\mathcal{K}$  if and only if  $\mathcal{A}^{Int} = \mathcal{A}$  and  $\mathcal{A} = (\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A)$  is a SNCS in  $\mathcal{K}$  if and only if  $\mathcal{A}^{Clo} = \mathcal{A}$ .

**Proposition 5.** Let  $\mathcal{A}$  be a SNS in  $\mathcal{K}$ . Then, following results hold for  $\mathcal{A}$ :

- (i)  $(1_{SN})^{Int} = 1_{SN}$ .
- (ii)  $(\emptyset_{SN})^{Clo} = \emptyset_{SN}$ .
- (iii)  $\overline{(\mathcal{A})}^{Int} = \overline{(\mathcal{A})}^{Clo}$ .
- (iv)  $\overline{(\mathcal{A})}^{Clo} = \overline{(\mathcal{A})}^{Int}$ .

**Definition 16.** Let  $\mathcal{K}$  be a K-algebra and  $\chi$  be a SNT on  $\mathcal{K}$ . A SNOS  $\mathcal{A}$  in  $\mathcal{K}$  is said to be single-valued neutrosophic regular open if

$$\mathcal{A} = (\mathcal{A}^{Clo})^{Int}. \quad (4)$$

**Remark 2.** Every SNOS which is regular is single-valued neutrosophic open and every single-valued neutrosophic closed and open set is a single-valued neutrosophic regular open.

**Definition 17.** A single-valued neutrosophic super connected K-algebra is such a K-algebra in which there does not exist a single-valued neutrosophic regular open set  $\mathcal{A} = (\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A)$  such that  $\mathcal{A} \neq \emptyset_{SN}$  and  $\mathcal{A} \neq 1_{SN}$ . If there exists such a single-valued neutrosophic regular open set  $\mathcal{A} = (\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A)$  such that  $\mathcal{A} \neq \emptyset_{SN}$  and  $\mathcal{A} \neq 1_{SN}$ , then K-algebra is said to be a single-valued neutrosophic super disconnected.

**Example 5.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a K-algebra, where  $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$  is the cyclic group of order 9 and Caley's table for  $\odot$  is given in Example 1 We define a SNS as:

$$\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}.$$

Let  $\chi_K = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$  be a SNT on  $\mathcal{K}$  and let  $\mathcal{B} = \{(e, 0.3, 0.3, 0.8), (s, 0.2, 0.2, 0.6)\}$  be a SNS in  $\mathcal{K}$ . here

SNOSs :  $\emptyset_{SN} = \{0, 0, 1\}, 1_{SN} = \{1, 1, 0\}, \mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$ .

SNCSs :  $(\emptyset_{SN})^c = (\{0, 0, 1\})^c = (\{1, 1, 0\}) = 1_{SN}, (1_{SN})^c = (\{1, 1, 0\})^c = (\{0, 0, 1\}) = \emptyset_{SN},$

$(\mathcal{A})^c = (\{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\})^c = (\{(e, 0.8, 0.3, 0.2), (s, 0.6, 0.2, 0.1)\}) = \mathcal{A}'$  (say).

Then, closure of  $\mathcal{B}$  is the intersection of closed sets which contain  $\mathcal{B}$ . Therefore,

$$\mathcal{A}' = \mathcal{B}^{Clo}. \quad (5)$$

Now, interior of  $\mathcal{B}$  is the union of open sets which contain in  $\mathcal{B}$ . Therefore,

$$\begin{aligned} \emptyset_{SN} \cup \mathcal{A} &= \mathcal{A} \\ \mathcal{A} &= \mathcal{B}^{Int}. \end{aligned} \quad (6)$$

Note that  $(\mathcal{B}^{Clo})^{Clo} = \mathcal{B}^{Clo}$ . Now, if we consider a SNS  $\mathcal{A} = \{(e, 0.2, 0.3, 0.8), (s, 0.1, 0.2, 0.6)\}$  in a  $K$ -algebra  $\mathcal{K}$  and if  $\chi_{\mathcal{K}} = \{\emptyset_{SN}, 1_{SN}, \mathcal{A}\}$  is a SNT on  $\mathcal{K}$ . Then,  $(\mathcal{A})^{Clo} = \mathcal{A}$  and  $(\mathcal{A})^{Int} = \mathcal{A}$ . Consequently,

$$\mathcal{A} = (\mathcal{A}^{Clo})^{Int}, \quad (7)$$

which shows that  $\mathcal{A}$  is a SN regular open set in  $K$ -algebra  $\mathcal{K}$ . Since  $\mathcal{A}$  is a SN regular open set in  $\mathcal{K}$  and  $\mathcal{A} \neq \emptyset_{SN}, \mathcal{A} \neq 1_{SN}$ , then, by Definition 17,  $K$ -algebra  $\mathcal{K}$  is a single-valued neutrosophic super disconnected  $K$ -algebra.

**Proposition 6.** Let  $\mathcal{K}$  be a  $K$ -algebra and let  $\mathcal{A}$  be a SNOS. Then, the following statements are equivalent:

- (i) A  $K$ -algebra is single-valued neutrosophic super connected.
- (ii)  $(\mathcal{A})^{Clo} = 1_{SN}$ , for each SNOS  $\mathcal{A} \neq \emptyset_{SN}$ .
- (iii)  $(\mathcal{A})^{Int} = \emptyset_{SN}$ , for each SNCS  $\mathcal{A} \neq 1_{SN}$ .
- (iv) There do not exist SNOSs  $\mathcal{A}, \mathcal{F}$  such that  $\mathcal{A} \subseteq \overline{\mathcal{F}}$  and  $\mathcal{A} \neq \emptyset_{SN} \neq \mathcal{F}$  in  $K$ -algebra  $\mathcal{K}$ .

**Definition 18.** Let  $(\mathcal{K}, \chi)$  be a SNTS, where  $\mathcal{K}$  is a  $K$ -algebra. Let  $S$  be a collection of SNOSs in  $\mathcal{K}$  denoted by  $S = \{(\mathcal{T}_{\mathcal{A}_j}, \mathcal{I}_{\mathcal{A}_j}, \mathcal{F}_{\mathcal{A}_j}) : j \in J\}$ . Let  $\mathcal{A}$  be a SNOS in  $\mathcal{K}$ . Then,  $S$  is called a single-valued neutrosophic open covering of  $\mathcal{A}$  if  $\mathcal{A} \subseteq \bigcup S$ .

**Definition 19.** Let  $\mathcal{K}$  be a  $K$ -algebra and  $(\mathcal{K}, \chi)$  be a SNTS. Let  $L$  be a finite sub-collection of  $S$ . If  $L$  is also a single-valued neutrosophic open covering of  $\mathcal{A}$ , then it is called a finite sub-covering of  $S$  and  $\mathcal{A}$  is called single-valued neutrosophic compact if each single-valued neutrosophic open covering  $S$  of  $\mathcal{A}$  has a finite sub-cover. Then,  $(\mathcal{K}, \chi)$  is called compact  $K$ -algebra.

**Remark 3.** If either  $\mathcal{K}$  is a finite  $K$ -algebra or  $\chi$  is a finite topology on  $\mathcal{K}$ , i.e., consists of finite number of single-valued neutrosophic subsets of  $\mathcal{K}$ , then the SNT  $(\mathcal{K}, \chi)$  is a single-valued neutrosophic compact topological space.

**Proposition 7.** Let  $(\mathcal{K}_1, \chi_1)$  and  $(\mathcal{K}_2, \chi_2)$  be two SNTSs and  $f$  be a single-valued neutrosophic continuous mapping from  $\mathcal{K}_1$  into  $\mathcal{K}_2$ . Let  $\mathcal{A}$  be a SNS in  $(\mathcal{K}_1, \chi_1)$ . If  $\mathcal{A}$  is single-valued neutrosophic compact in  $(\mathcal{K}_1, \chi_1)$ , then  $f(\mathcal{A})$  is single-valued neutrosophic compact in  $(\mathcal{K}_2, \chi_2)$ .

**Proof.** Let  $f : (\mathcal{K}_1, \chi_1) \rightarrow (\mathcal{K}_2, \chi_2)$  be a single-valued neutrosophic continuous function. Let  $\acute{S} = (f^{-1}(\mathcal{A}_j : j \in J))$  be a single-valued neutrosophic open covering of  $\mathcal{A}$  since  $\mathcal{A}$  be a SNS in  $(\mathcal{K}_1, \chi_1)$ . Let  $\acute{L} = (\mathcal{A}_j : j \in J)$  be a single-valued neutrosophic open covering of  $f(\mathcal{A})$ . Since  $\mathcal{A}$  is compact, then there exists a single-valued neutrosophic finite sub-cover  $\bigcup_{j=1}^n f^{-1}(\mathcal{A}_j)$  such that

$$\mathcal{A} \subseteq \bigcup_{j=1}^n f^{-1}(\mathcal{A}_j)$$

We have to prove that there also exists a finite sub-cover of  $\acute{L}$  for  $f(\mathcal{A})$  such that

$$f(\mathcal{A}) \subseteq \bigcup_{j=1}^n (\mathcal{A}_j)$$

Now,

$$\begin{aligned}\mathcal{A} &\subseteq \bigcup_{j=1}^n f^{-1}(\mathcal{A}_j) \\ f(\mathcal{A}) &\subseteq f\left(\bigcup_{j=1}^n f^{-1}(\mathcal{A}_j)\right) \\ f(\mathcal{A}) &\subseteq \bigcup_{j=1}^n (f(f^{-1}(\mathcal{A}_j))) \\ f(\mathcal{A}) &\subseteq \bigcup_{j=1}^n (\mathcal{A}_j).\end{aligned}$$

Hence,  $f(\mathcal{A})$  is single-valued neutrosophic compact in  $(\mathcal{K}_2, \chi_2)$ .  $\square$

**Definition 20.** A single-valued neutrosophic set  $\mathcal{A}$  in a  $K$ -algebra  $\mathcal{K}$  is called a single-valued neutrosophic point if

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(v) &= \begin{cases} \alpha \in (0, 1], & \text{if } v=u \\ 0, & \text{otherwise,} \end{cases} \\ \mathcal{I}_{\mathcal{A}}(v) &= \begin{cases} \beta \in (0, 1], & \text{if } v=u \\ 0, & \text{otherwise,} \end{cases} \\ \mathcal{F}_{\mathcal{A}}(v) &= \begin{cases} \gamma \in [0, 1), & \text{if } v=u \\ 0, & \text{otherwise,} \end{cases}\end{aligned}$$

with support  $u$  and value  $(\alpha, \beta, \gamma)$ , denoted by  $u(\alpha, \beta, \gamma)$ . This single-valued neutrosophic point is said to “belong to” a SNS  $\mathcal{A}$ , written as  $u(\alpha, \beta, \gamma) \in \mathcal{A}$  if  $\mathcal{T}_{\mathcal{A}}(u) \geq \alpha, \mathcal{I}_{\mathcal{A}}(u) \geq \beta, \mathcal{F}_{\mathcal{A}}(u) \leq \gamma$  and said to be “quasi-coincident with” a SNS  $\mathcal{A}$ , written as  $u(\alpha, \beta, \gamma) q \mathcal{A}$  if  $\mathcal{T}_{\mathcal{A}}(u) + \alpha > 1, \mathcal{I}_{\mathcal{A}}(u) + \beta > 1, \mathcal{F}_{\mathcal{A}}(u) + \gamma < 1$ .

**Definition 21.** Let  $\mathcal{K}$  be a  $K$ -algebra and let  $(\mathcal{K}, \chi)$  be a SNTS. Then,  $(\mathcal{K}, \chi)$  is called a single-valued neutrosophic Hausdorff space if and only if, for any two distinct single-valued neutrosophic points  $u_1, u_2 \in \mathcal{K}$ , there exist SNOs  $\mathcal{B}_1 = (\mathcal{T}_{\mathcal{B}_1}, \mathcal{I}_{\mathcal{B}_1}, \mathcal{F}_{\mathcal{B}_1}), \mathcal{B}_2 = (\mathcal{T}_{\mathcal{B}_2}, \mathcal{I}_{\mathcal{B}_2}, \mathcal{F}_{\mathcal{B}_2})$  such that  $u_1 \in \mathcal{B}_1, u_2 \in \mathcal{B}_2$ , i.e.,

$$\begin{aligned}\mathcal{T}_{\mathcal{B}_1}(u_1) &= 1, \mathcal{I}_{\mathcal{B}_1}(u_1) = 1, \mathcal{F}_{\mathcal{B}_1}(u_1) = 0, \\ \mathcal{T}_{\mathcal{B}_2}(u_2) &= 1, \mathcal{I}_{\mathcal{B}_2}(u_2) = 1, \mathcal{F}_{\mathcal{B}_2}(u_2) = 0\end{aligned}$$

and satisfy the condition that  $\mathcal{B}_1 \cap \mathcal{B}_2 = \emptyset_{\text{SN}}$ . Then,  $(\mathcal{K}, \chi)$  is called single-valued neutrosophic Hausdorff space and  $K$ -algebra is said to be a Hausdorff  $K$ -algebra. In fact,  $(\mathcal{K}, \chi)$  is a Hausdorff  $K$ -algebra.

**Example 6.** Let  $\mathcal{K} = (G, \cdot, \odot, e)$  be a  $K$ -algebra and let  $(\mathcal{K}, \chi_{\mathcal{K}})$  be a SNTS on  $\mathcal{K}$ , where  $G = \{e, x, x^2, x^3, x^4, x^5, x^6, x^7, x^8\}$  is the cyclic group of order 9 and Caley’s table for  $\odot$  is given in Example 1. We define two SNSs as  $\mathcal{A} = \{(e, 1, 1, 0), (s, 0, 0, 1)\}, \mathcal{B} = \{(e, 0, 0, 1), (s, 1, 1, 0)\}$ . Consider a single-valued neutrosophic point for  $e \in \mathcal{K}$  such that

$$\begin{aligned}\mathcal{T}_{\mathcal{A}}(e) &= \begin{cases} 0.3, & \text{if } e=u \\ 0, & \text{otherwise,} \end{cases} \\ \mathcal{I}_{\mathcal{A}}(e) &= \begin{cases} 0.2, & \text{if } e=u \\ 0, & \text{otherwise,} \end{cases}\end{aligned}$$

$$\mathcal{F}_A(e) = \begin{cases} 0.4, & \text{if } e=u \\ 0, & \text{otherwise.} \end{cases}$$

Then,  $e(0.3, 0.2, 0.4)$  is a single-valued neutrosophic point with support  $e$  and value  $(0.3, 0.2, 0.4)$ . This single-valued neutrosophic point belongs to SNS "A" but not SNS "B".

Now, for all  $s \neq e \in \mathcal{K}$

$$\mathcal{T}_B(s) = \begin{cases} 0.5, & \text{if } s=u \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathcal{I}_B(s) = \begin{cases} 0.4, & \text{if } s=u \\ 0, & \text{otherwise,} \end{cases}$$

$$\mathcal{F}_B(s) = \begin{cases} 0.3, & \text{if } s=u \\ 0, & \text{otherwise.} \end{cases}$$

Then,  $s(0.5, 0.4, 0.3)$  is a single-valued neutrosophic point with support  $s$  and value  $(0.5, 0.4, 0.3)$ . This single-valued neutrosophic point belongs to SNS "B" but not SNS "A". Thus,  $e(0.3, 0.2, 0.4) \in \mathcal{A}$  and  $e(0.3, 0.2, 0.4) \notin \mathcal{B}$ ,  $s(0.5, 0.4, 0.3) \in \mathcal{B}$  and  $s(0.5, 0.4, 0.3) \notin \mathcal{A}$  and  $\mathcal{A} \cap \mathcal{B} = \emptyset_{SN}$ . Thus,  $\mathcal{K}$ -algebra is a Hausdorff  $\mathcal{K}$ -algebra and  $(\mathcal{K}, \chi_{\mathcal{K}})$  is a Hausdorff topological space.

**Theorem 5.** Let  $(\mathcal{K}_1, \chi_1), (\mathcal{K}_2, \chi_2)$  be two SNTSs. Let  $f$  be a single-valued neutrosophic homomorphism from  $(\mathcal{K}_1, \chi_1)$  into  $(\mathcal{K}_2, \chi_2)$ . Then,  $(\mathcal{K}_1, \chi_1)$  is a single-valued neutrosophic Hausdorff space if and only if  $(\mathcal{K}_2, \chi_2)$  is a single-valued neutrosophic Hausdorff  $\mathcal{K}$ -algebra.

**Proof.** Let  $(\mathcal{K}_1, \chi_1), (\mathcal{K}_2, \chi_2)$  be two SNTSs. Let  $\mathcal{K}_1$  be a single-valued neutrosophic Hausdorff space, then, according to the Definition 21, there exist two SNOs  $X$  and  $Y$  for two distinct single-valued neutrosophic points  $u_1, u_2 \in \chi_2$  also  $a, b \in \mathcal{K}_1 (a \neq b)$  such that  $X \cap Y = \emptyset_{SN}$ . Now, for  $w \in \mathcal{K}_1$ , consider  $(f^{-1}(u_1))(w) = u_1(f^{-1}(w))$ , where  $u_1(f^{-1}(w)) = s \in (0, 1]$  if  $w = f^{-1}(a)$ , otherwise 0. That is,  $(f^{-1}(u_1))(w) = ((f^{-1}(u))_1(w))$ . Therefore, we have  $f^{-1}(u_1) = (f^{-1}(u))_1$ . Similarly,  $f^{-1}(u_2) = (f^{-1}(u))_2$ . Now, since  $f^{-1}$  is a single-valued neutrosophic continuous mapping from  $\mathcal{K}_2$  into  $\mathcal{K}_1$ , there exist two SNOs  $f(X)$  and  $f(Y)$  of  $u_1$  and  $u_2$ , respectively, such that  $f(X) \cap f(Y) = f(\emptyset_{SN}) = \emptyset_{SN}$ . This implies that  $\mathcal{K}_2$  is a single-valued neutrosophic Hausdorff  $\mathcal{K}$ -algebra. The converse part can be proved similarly.  $\square$

**Theorem 6.** Let  $f$  be a single-valued neutrosophic continuous function which is both one-one and onto, where  $f$  is a mapping from a single-valued neutrosophic compact  $\mathcal{K}$ -algebra  $\mathcal{K}_1$  into a single-valued neutrosophic Hausdorff  $\mathcal{K}$ -algebra  $\mathcal{K}_2$ . Then,  $f$  is a homomorphism.

**Proof.** Let  $f : \mathcal{K}_1 \rightarrow \mathcal{K}_2$  be a single-valued neutrosophic continuous bijective function from single-valued neutrosophic compact  $\mathcal{K}$ -algebra  $\mathcal{K}_1$  into a single-valued neutrosophic Hausdorff  $\mathcal{K}$ -algebra  $\mathcal{K}_2$ . Since  $f$  is a single-valued neutrosophic continuous mapping from  $\mathcal{K}_1$  into  $\mathcal{K}_2$ ,  $f$  is a homomorphism. Since  $f$  is bijective, we only prove that  $f$  is single-valued neutrosophic closed. Let  $\mathcal{D} = (\mathcal{T}_{\mathcal{D}}, \mathcal{I}_{\mathcal{D}}, \mathcal{F}_{\mathcal{D}})$  be a single-valued neutrosophic closed in  $\mathcal{K}_1$ . If  $\mathcal{D} = \emptyset_{SN}$  is single-valued neutrosophic closed in  $\mathcal{K}_1$ , then  $f(\mathcal{D}) = \emptyset_{SN}$  is single-valued neutrosophic closed in  $\mathcal{K}_2$ . However, if  $\mathcal{D} \neq \emptyset_{SN}$ , then  $\mathcal{D}$  will be a single-valued neutrosophic compact, being subset of a single-valued neutrosophic compact  $\mathcal{K}$ -algebra. Then,  $f(\mathcal{D})$ , being single-valued neutrosophic continuous image of a single-valued neutrosophic compact  $\mathcal{K}$ -algebra, is also single-valued neutrosophic compact. Therefore,  $\mathcal{K}_2$  is closed, which implies that mapping  $f$  is closed. Thus,  $f$  is a homomorphism.  $\square$

#### 4. Conclusions

Non-classical logic is considered as a powerful tool for inspecting uncertainty and indeterminacy found in real world problems. Being a great extension of classical logic, neutrosophic set theory is considered as a useful mathematical tool to cope up with uncertainties in science, technology, and computer science. We have used this mathematical model with a topological structure to investigate the uncertainty in  $K$ -algebras. We have introduced the notion of single-valued neutrosophic topological  $K$ -algebras and presented certain concepts, including continuous function between two topological on  $K$ -algebras, relatively continuous function and homomorphism. We have investigated the image and pre-image of single-valued neutrosophic topological  $K$ -algebras under this homomorphism. We have proposed some conclusive concepts, including single-valued neutrosophic compact  $K$ -algebras and single-valued neutrosophic Hausdorff  $K$ -algebras. We plan to extend our study to: (i) single-valued neutrosophic soft topological  $K$ -algebras; and (ii) bipolar neutrosophic soft topological  $K$ -algebras.

For other notations and terminologies, readers are referred to [21–26].

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#### References

1. Dar, K.H.; Akram, M. On a  $K$ -algebra built on a group. *Southeast Asian Bull. Math.* **2005**, *29*, 41–49.
2. Dar, K.H.; Akram, M. Characterization of a  $K(G)$ -algebra by self maps. *Southeast Asian Bull. Math.* **2004**, *28*, 601–610.
3. Dar, K.H.; Akram, M. On  $K$ -homomorphisms of  $K$ -algebras. *Int. Math. Forum* **2007**, *46*, 2283–2293.
4. Akram, M.; Dar, K.H.; Jun, Y.B.; Roh, E.H. Fuzzy structures of  $K(G)$ -algebra. *Southeast Asian Bull. Math.* **2007**, *31*, 625–637.
5. Akram, M.; Dar, K.H. *Generalized Fuzzy  $K$ -Algebras*; VDM Verlag: Saarbrücken, Germany, 2010; p. 288, ISBN 978-3-639-27095-2.
6. Smarandache, F. *Neutrosophy Neutrosophic Probability, Set, and Logic*; Amer Res Press: Rehoboth, MA, USA, 1998.
7. Atanassov, K. Intuitionistic fuzzy sets. *Fuzzy Sets Syst.* **1986**, *20*, 87–96.
8. Wang, H.; Smarandache, F.; Zhang, Y.Q.; Sunderraman, R. Single valued neutrosophic sets. *Multispace Multistruct* **2010**, *4*, 410–413.
9. Agboola, A.A.A.; Davvaz, B. Introduction to neutrosophic  $BCI/BCK$ -algebras. *Int. J. Math. Math. Sci.* **2015**, *6*, doi:10.1155/2015/370267.
10. June, Y.B. Neutrosophic subalgebras of several types in  $BCK/BCK$ -algebras. *Annl. Fuzzy Math. Inform.* **2017**, *14*, 75–86.
11. June, Y.B.; Kim, S.J.; Smarandache, F. Interval neutrosophic sets with applications in  $BCK/BCK$ -algebra. *Axioms* **2018**, *7*, 23, doi:10.3390/axioms7020023.
12. Jun, Y.B.; Smarandache, F.; Song, S.Z.; Khan, M. Neutrosophic positive implicative  $N$ -ideals in  $BCK$ -algebras. *Axioms* **2018**, *7*, 3, doi:10.3390/axioms7010003.
13. Chang, C.L. Fuzzy topological spaces. *J. Math. Anal. Appl.* **1968**, *24*, 182–190.
14. Lowen, R. Fuzzy topological spaces and fuzzy compactness. *J. Math. Anal. Appl.* **1976**, *56*, 621–633, doi:10.1016/0022-247X(76)90029-9.
15. Pu, P.M.; Liu, Y.M. Fuzzy topology, I. Neighbourhood structure of a fuzzy point and Moore-Smith convergence. *J. Math. Anal. Appl.* **1980**, *76*, 571–599.
16. Chattopadhyay, K.C.; Samanta, S.K. Fuzzy topology: Fuzzy closure operator, fuzzy compactness and fuzzy connectedness. *Fuzzy Sets Syst.* **1993**, *54*, 207–212, doi:10.1016/0165-0114(93)90277-O.

17. Coker, D. An introduction to intuitionistic fuzzy topological spaces. *Fuzzy Sets Syst.* **1997**, *88*, 81–89, doi:10.1016/S0165-0114(96)00076-0.
18. Salama, A.A.; Alblowi, S.A. Neutrosophic set and neutrosophic topological spaces. *IOSR-JM* **2012**, *3*, 31–35, doi:10.9790/5728-0343135.
19. Akram, M.; Dar, K.H. On fuzzy topological  $K$ -algebras. *Int. Math. Forum* **2006**, *23*, 1113–1124.
20. Akram, M.; Dar, K.H. Intuitionistic fuzzy topological  $K$ -algebras. *J. Fuzzy Math.* **2009**, *17*, 19–34.
21. Lupianez, F.G. Hausdorffness in intuitionistic fuzzy topological spaces. *Mathw. Soft Comput.* **2003**, *10*, 17–22.
22. Hanafy, I.M. Completely continuous functions in intuitionistic fuzzy topological spaces. *Czechoslovak Math. J.* **2003**, *53*, 793–803, doi:10.1023/B:CMAJ.0000024523.64828.31.
23. Jun, Y.B.; Song, S.Z.; Smarandache, F.; Bordbar, H. Neutrosophic quadruple  $BCK/BCI$ -algebras. *Axioms* **2018**, *7*, 41, doi:10.3390/axioms7020041.
24. Elias, J.; Rossi, M.E. The structure of the inverse system of Gorenstein  $K$ -algebras. *Adv. Math.* **2017**, *314*, 306–327, doi:10.1016/j.aim.2017.04.025.
25. Masuti, S.K.; Tozzo, L. The structure of the inverse system of level  $K$ -algebras. *Collect. Math.* **2017**, *1–27*, doi:10.1007/s13348-018-0212-3.
26. Borzooei, R.; Zhang, X.; Smarandache, F.; Jun, Y. Commutative generalized neutrosophic ideals in  $BCK$ -algebras. *Symmetry* **2018**, *10*, 350, doi:10.3390/sym10080350.



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